No. V.— Cardan's Rings.

We take this puzzle next in order, as having a close affinity in principle with the problem of Brahma, which precedes it. It is one of the oldest of known puzzles, having been learnedly discussed by the mathematician, Jerome Cardan, as early as the sixteenth century. Whether it was his own invention is doubtful, but it was for many generations associated with his name. At the present day people have forgotten all about Cardan, and the problem is now more frequently referred to by the less distinctive title of "The Puzzling Rings." In French it is known as *La Baguenaudier*, and it is said to be now and then found on an enlarged scale in English rural districts, forged in iron, and appropriately called "The Tiring Irons." It has more than once been deemed worthy of notice by mathematicians, the learned Savilian professor, Dr. Wallis, devoting to it a special section of his treatise on Algebra (1685), under the title *De Complicatis Annulis*.

The apparatus consists of four parts :—

(1) A wire bow or shuttle (sometimes provided with a handle at one end).

(2) A flat bar of wood, metal, or bone, a trifle larger than the bow, with holes through it at regular intervals, corresponding in number with the rings.

(3) A number (six to twelve, as the case may be), of rings, which should in internal diameter be just double the external width, and in thickness one-third of the internal width of the bow.

(4) A series of short wires, corresponding in number with that of the rings.

One end of each wire passes through one of the holes in the bar, and is rivetted on the opposite side, though the hole is of such a size as to allow it free play. It thence passes through the bow, and through one of the rings, and its oppo-

site end is then bent round another ring, the result being as shown in Fig. 454 (representing the 10-ring form of the puzzle). The rings are all threaded on the bow, each (with one exception) passing around the wire of its right-hand
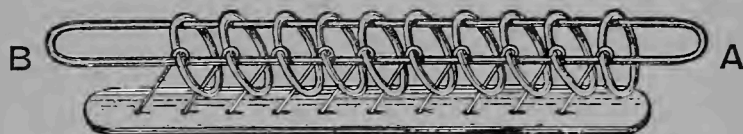

FIG. 454.

neighbour. The exception is the ring to the extreme right, which, having no neighbour on its outer side, enjoys a degree of liberty not shared by the remaining rings. The puzzle is to get the rings off the bow.

It should be mentioned that, as in the case of the Brahmin Puzzle, each additional ring doubles the time occupied in the solution. With seven rings, the puzzle requires 85 moves to solve it ; with eight rings, 170 ; with nine, 341 ; and with ten, 682. With eleven it would require 1,365 movements, and with twelve, 2,730. Ten rings are the popular limit, and we have therefore selected the puzzle in this form for illustration.*

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* The solution given in the Key will equally apply to the case of any smaller number of rings, the only difference being that the process will be cut short at an earlier stage.

No. V.—Cardan's Rings.

We quote the following instructions, the clearest and most practical that we have yet seen for the solution for this puzzle, from an anonymous American writer.†

"Take the bow in your left hand, holding it at the end, *B*, and consider the rings as being numbered 1 to 10. The first will be the extreme ring to the right, and the tenth the nearest to your left hand. (See Fig. 454.)

"It will be seen that the difficulty arises from each ring passing round the wire of its right-hand neighbour. The extreme ring at the right hand, of course, being unconnected with any other wire than its own, may at any time be drawn off the end of the bow at *A*, raised up, dropped through the bow, and finally released. After you have done this, try to pass the second ring in the same way, and you will not succeed, as it is obstructed by the wire of the first ring; but if you bring the first ring on again, by reversing the process by which you took it off,—viz., by putting it up through the bow and on to the end of it,—you will then find that by taking the first and second rings together they will both draw off, lift up, and drop through the bow. Having done this, try to pass the third ring off, and you will not be able, because it is fastened on one side to its own wire, which is within

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† For another very clever explanation of this puzzle see the *Ency-clopédie Méthodique des Jeux*, pp. 424 *et seq*.

the bow, and on the other side to the second ring, which is without the bow.

" Therefore, leaving the third ring for the present, try the fourth ring, which is now at the end all but one, and both of the wires which affect it being within the bow, you will draw it off without obstruction. In doing this, you will have to slip the third ring off, which will not drop through for the reasons before given ; so, having dropped the fourth ring through, you can only slip the third ring on again. You will now comprehend that (with the exception of the first ring) the only ring which can at any time be released is that which happens to be *second on the bow, at the right-hand end*; because both the wires which affect it being within the bow, there will be no impediment to its dropping through.

" You have now the first and second rings released, and the fourth also, the third still fixed, to release which we must make it last but one on the bow. To effect this, pass the first and second rings together through the bow, and on to it ; then release the first ring again by slipping it off and dropping it through, and the third ring will stand as second on the bow, in its proper position for releasing, by drawing the second and third off altogether, dropping the third through, and slipping the second on again. Now, to release the second, put the first up, through and on the bow, then slip the two together off, raise them up, and drop them through.

" The sixth will now stand second, consequently in its proper position for releasing, therefore draw it towards the end, *A* ; slip the fifth off, then the sixth, and drop it through ; after which, replace the fifth, as you cannot release it until it stands in the position of a second ring. In order to effect this, you must bring the first and second rings together, through and on to the bow ; then, in order to get the third on, slip the first off, and down through the bow ; then bring the third up, through and on to the bow, then bring the first ring up and on again, and releasing the first and second together, bring the fourth through and on to the bow, replacing the third. Then bring the first and second together on, drop the first off and through, then the third the same ; replace the first on the bow, take off the first and second together, and the fifth will then stand second, as you desired ; draw it towards the end, slip it off and through, replace the fourth, bring the first and second together up and on again ;

release the first, bring on the third, passing the second ring on to the bow again; replace the first, in order to release the first and second together, then bring the fourth toward the end, slipping it off and through; replace the third, bring the first and second together up and on again, release the first, then the third, replacing the second, bring the first up and on, in order to release the first and second together, which having done, your eighth ring will then stand second; consequently you can release it, slipping the seventh on again.

"To release the seventh, you must begin by putting the first and second up and on together, and going through the movements in the same succession as before, until you find you have only the tenth and ninth on the bow; then slip the tenth off and through the bow, and replace the ninth. This dropping of the tenth ring is the first effectual movement toward getting the rings off, as all the changes you have gone through were only to enable you to get at the tenth ring.

"You will now find that you have only the ninth left on the bow, and you must not be discouraged on learning that in order to get that ring off, all the others to the right hand must be put on again, beginning by putting the first and second together, and working as before, until you find that the ninth stands as second on the bow, at which time you can release it. You will then have only the eighth left on the bow; you must again put on all the rings to the right hand, beginning by putting up the first and second together, till you find the eighth standing as second on the bow, or in its proper position for releasing, and so you proceed until you find all the rings finally released.

"As you commence your operations with all the rings ready fixed on the bow, you will release the tenth ring in 170 moves; but as you then have only the ninth on, and as it is necessary to bring on again all the rings up to the ninth, in order to release the ninth (which requires sixteen moves more), you will, consequently, release the ninth ring in 256 moves. For your encouragement, your labour will diminish by one half with each following ring which is finally released. The eighth comes off in 128 moves. The seventh in 64 moves, and so on, until you arrive at the second and first rings, which come off together, making 681 moves, which are necessary to take off all the rings.

" With the experience you will by this time have acquired, it is only necessary to say that, to replace the rings, you begin by putting up the first and second together, and follow precisely the same system as before, in reverse order. "