

Abstract

The blues progression is undoubtedly the most famous chord progression in modern music. But why is this harmonic sequence so perfect? This paper suggests an answer to the universal and eternal thrill of the blues via roller coasters and a special vertex colouring of the seven cycle.

There is no chord progression quite like the 12 bar blues. It forms the basis of what is classic rock n roll. Chuck Berry's *Johnny B. Goode*, Elvis Presley's *Hound Dog*, Little Richard's *Lucille* and *Long Tall Sally*, The Beatles' *The Word*, Led Zeppelin's *Rock n Roll*, Loggins and Messina's *Your Mama Don't Dance*, and classics like *Rock Around the Clock*, *Dizzie Miss Lizzie*, *Bad Boy* and *Kansas City* are all, at heart, blues. And songs like *Day Tripper* jump out in the way they break out of the blues that they set up so carefully.

Here is the blues chord progression that I have most often come across, and the one I find as the most compelling: the tonic chord (that is, chord I in the key) lasts for four bars (each consisting of four beats), followed by chord IV (the *subdominant*, on the fourth of the key) for two bars, followed by chord I for two bars, and then chords V (the *dominant*, on the fifth of the key), IV, I and V, each for one bar:

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I   / / / | / / / / | / / / / | / / / / |
IV  / / / | / / / / | I / / / | / / / / |
V   / / / | IV / / / | I / / / | V / / / |
    
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The blues progression has a variety of minor modifications, with sevenths (or ninths) often added to chords to increase tension, and occasionally, the chord in bar 12 staying on the tonic I (though I find such blues lacks the drive to repeat that those ending on the dominant V do). Here is what a basic blues in E (a common key for guitarists) might look like in music notation:

Figure 1: A Basic Blues in E

Like its masters, the blues just gets better with age. To paraphrase B.B. King, the thrill is never gone. I have yet to see any reasoning why the blues are the way they are: Why are they 12 bars long? Why do they have the chords they have? Why do the chords last as long as they do? And why can the chord progression be cycled seemingly endlessly without losing listener interest? The feeling that the blues chord progression generates is so powerful, so eternal, so right, that I sensed that there must be some underlying mathematics. And here is what I think is the mathematics behind the beauty of the blues.

Remember how exciting it was to go on a roller coaster? Up and down, peak to valley to peak again. The tracks were laid out to maximize the tension and release. Now, how can I create a chord progression, that is, a sequence of chords, that matches the thrill of a ride on a roller coaster?

In constructing a roller coaster, I would want to have three equal sections for the ride - the beginning, middle and end. I also want the excitement to build from one section to the next. The easiest way to accelerate is to double the sense of movement from one section to the next. So if the chord progression

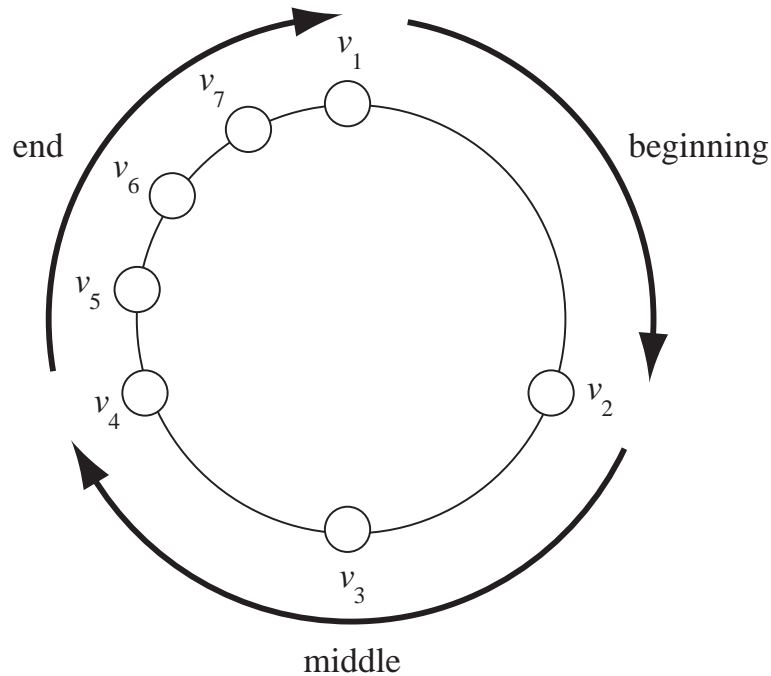


Figure 2: The Roller Coaster

stays on one chord for the beginning section, it should have two chords equally spaced for the middle and four equally spaced chords for the finish. Working backwards, if we take the smallest segment for a chord change to be one bar, then we should make the finish four bars, each with a different chord, the middle with two chords, each lasting two bars, and the beginning part with just one chord lasting for four bars.

What do we have in total? Twelve bars for our musical roller coaster. The sequence of seven chords in the harmonic progression that cycle around can be viewed as the graph C_7 . Any assignment of chords to the progression correspond to a vertex colouring of the graph, as adjacent chords in the progression need to be different. Thus we see immediately from the fact that the seven cycle C_7 has chromatic number 3 that we will need at least three chords in our roller coaster chord progression, and indeed three chords (our "colours") will suffice. The most basic chords in any key are the I, IV and V chords, so we take these as our colours for our C_7 chord progression. We note that tension-wise, the I chord provides the least tension (it is the root chord of the key), while the V chord provides the most – not all colours are created equal here.

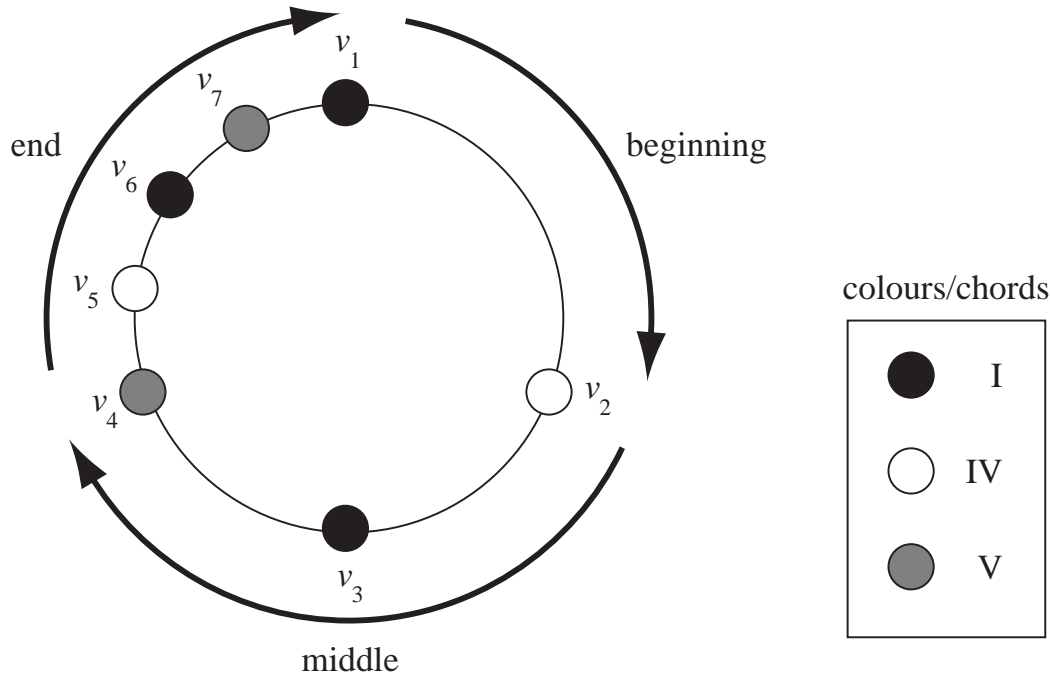


Figure 3: The Roller Coaster 3-Colouring of C_7

Now let's talk more about the shape of the roller coaster. We'll start off at base level; musically, this would be the tonic chord, that is, the major chord that defines the key. Thus vertex v_1 gets colour I. Vertex v_2 is our first climb (in the middle section of the roller coaster), and we should save the most exciting colour (V) for the end of the ride. That means v_2 and v_3 must be coloured IV and I, respectively.

Now for the finish. We save our biggest climb for the beginning of the finish, where we jump up to chord V from chord 4, that is, we use our most exciting colour, V, for v_4 . A good roller coaster would prolong our descent back down to earth, so chord 5 should not be the tonic I, which leaves v_5 to be coloured with IV. Dropping down to the ground, we see that v_6 should be coloured with the tonic chord, I. We save one great thrill for the end, by climbing back up to the top, with chord 7, so that the final vertex, v_7 , of our seven cycle chord progression, coloured with V. Our unique roller coaster colouring of C_7 is indeed the standard 12 bar blues, and having completed the ride, another can begin!

There are, of course, various alterations of the blues, such as inserting chord IV in bar 2, or removing the V chord in bar 12, but I view these as minor modifications of the basic roller coaster. Now of course there is more to a 12 bar blues than just the chords. The melody and instrumental parts all add to shape the roller coaster. I remember reading once that Paul McCartney's and John Lennon's goal was to write a simple rock song as good as Little Richard's *Long Tall Sally*. This song has always been one of my favourites, and it just throbs with excitement. If you look through the melody, you'll see how Little Richard, both by choice of notes and rhythm, increases the thrill of the roller coaster ride. During the first four bars of the verse, the chord remains the same. But Little Richard accelerates through these opening four bars by (almost) singing straight quarter notes for the first 2 bars, then doubling the feel by singing straight eighth notes for the next two bars: This gives us a mathematically defined "rush" to the first four bars (the long flat part of the initial part of the ride). If you look at the remaining melody, you see how the intervals for "baby" grow larger and larger, increasing the acceleration as we pass through to the highest point of the roller coaster in bar 9. It's interesting to note that in the Beatles' rendition of the song, Paul McCartney's bass line at the end of bar 12 of the verse climbs rather than falls back down to the tonic. This gives the illusion that we are climbing to yet a higher level, a seemingly "higher" tonic chord, even though we get back to where we start – an aural paradox within the roller coaster of the blues.

Acknowledgement

This article was partially supported by a grant from the Natural Sciences and Engineering Research Council of Canada.